CryptoVerif Tutorial

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Exercise 1: preliminary definition SUF-CMA

Definition (SUF-CMA MACs)

The advantage of the adversary against strong unforgeability under chosen message attacks (SUF-CMA) of MACs is:

\[
\text{Succ}_{\text{MAC}}^{\text{suf-cma}}(t, q_m, q_v, l) = \max_{A} \Pr \left[ k \leftarrow \text{mkgen}; (m, s) \leftarrow A^{\text{mac}(.,k),\text{verify}(.,k,.)} : \text{verify}(m, k, s) \land \text{no query to the oracle mac}(., k) \text{ with message } m \text{ returned } s \right] \]

where \( A \) runs in time at most \( t \),
calls \( \text{mac}(., k) \) at most \( q_m \) times with messages of length at most \( l \),
calls \( \text{verify}(., k, .) \) at most \( q_v \) times with messages of length at most \( l \).

MAC is SUF-CMA if and only if \( \text{Succ}_{\text{MAC}}^{\text{suf-cma}}(t, q_m, q_v, l) \) is negligible when \( t, q_m, q_v, l \) are polynomial in the security parameter.
Exercise 1: preliminary definition UF-CMA

Definition (UF-CMA MACs)

The advantage of the adversary against unforgeability under chosen message attacks (UF-CMA) of MACs is:

\[ \text{Succ}_{\text{MAC}}^{\text{uf-cma}}(t, q_m, q_v, l) = \]

\[
\max_A \Pr \left[ \begin{array}{l}
R \leftarrow \text{mkgen}; (m, s) \leftarrow A^{\text{mac}(., k), \text{verify}(., k, .)} : \text{verify}(m, k, s) \land \\
\text{m was never queried to the oracle mac}(., k)
\end{array} \right]
\]

where \( A \) runs in time at most \( t \),
calls \( \text{mac}(., k) \) at most \( q_m \) times with messages of length at most \( l \),
calls \( \text{verify}(., k, .) \) at most \( q_v \) times with messages of length at most \( l \).

MAC is UF-CMA if and only if \( \text{Succ}_{\text{MAC}}^{\text{uf-cma}}(t, q_m, q_v, l) \) is negligible when \( t, q_m, q_v, l \) are polynomial in the security parameter.
Exercise 1: preliminary definition IND-CCA2

Definition (IND-CCA2 symmetric encryption)

The advantage of the adversary against indistinguishability under adaptive chosen-ciphertext attacks (IND-CCA2) of a symmetric encryption scheme SE is:

\[
\text{Succ}_{\text{SE}}^{\text{ind-cca2}}(t, q_e, q_d, l_e, l_d) = \max_A 2 \Pr \left[ \begin{array}{c}
    b \leftarrow \{0, 1\}; k \leftarrow \text{kgen}; \\
    b' \leftarrow A^{\text{enc}(LR(\ldots, b), k), \text{dec}(\ldots, k)} : b' = b \land \\
    A \text{ has not called } \text{dec}(\ldots, k) \text{ on the result of } \\
    \text{enc}(LR(\ldots, b), k)
\end{array} \right] - 1
\]

where \( A \) runs in time at most \( t \), calls \( \text{enc}(LR(\ldots, b), k) \) at most \( q_e \) times on messages of length at most \( l_e \), calls \( \text{dec}(\ldots, k) \) at most \( q_d \) times on messages of length at most \( l_d \).

SE is IND-CCA2 if and only if \( \text{Succ}_{\text{SE}}^{\text{ind-cca2}}(t, q_e, q_d, l_e, l_d) \) is negligible when \( t, q_e, q_d, l_e, l_d \) are polynomial in the security parameter.
Exercise 1: preliminary definition INT-CTXT

Definition (INT-CTXT symmetric encryption)

The advantage of the adversary against ciphertext integrity (INT-CTXT) of a symmetric encryption scheme SE is:

\[
\text{Succ}_{SE}^{\text{int-ctxt}}(t, q_e, q_d, l_e, l_d) = \max_{\mathcal{A}} \Pr \left[ k \overset{R}{\leftarrow} kgen; c \leftarrow \mathcal{A}^{\text{enc}(.,k),\text{dec}(.,k)\neq \bot} : \text{dec}(c, k) \neq \bot \wedge \text{c is not the result of a call to the enc(., k) oracle} \right]
\]

where \(\mathcal{A}\) runs in time at most \(t\),
calls \(\text{enc}(., k)\) at most \(q_e\) times with messages of length at most \(l_e\),
calls \(\text{dec}(., k) \neq \bot\) at most \(q_d\) times with messages of length at most \(l_d\).

SE is **INT-CTXT** if and only if \(\text{Succ}_{SE}^{\text{int-ctxt}}(t, q_e, q_d, l_e, l_d)\) is negligible when \(t, q_e, q_d, L_e, l_d\) are polynomial in the security parameter.
Exercise 1

1. Show using CryptoVerif that, if the MAC scheme is probabilistic and SUF-CMA and the encryption scheme is IND-CPA, then the encrypt-then-MAC scheme is IND-CPA.

2. Show using the same assumptions that the encrypt-then-MAC scheme is IND-CCA2.

3. Show using the same assumptions that the encrypt-then-MAC scheme is INT-CTXT.

4. What happens if the MAC scheme is only UF-CMA?
Exercise 2: Preliminary definition

A public-key encryption scheme \( \text{AE} \) consists of

- a key generation algorithm \((pk, sk) \xleftarrow{R} \text{kgen}\)
- a probabilistic encryption algorithm \(\text{enc}(m, pk)\)
- a decryption algorithm \(\text{dec}(m, sk)\)

such that \(\text{dec}(\text{enc}(m, pk), sk) = m\).

The advantage of the adversary against indistinguishability under chosen-plaintext attacks (IND-CPA) is

\[
\text{Succ}^{\text{ind-cca2}}_{\text{AE}}(t) = \max_{A} 2 \Pr \left[ \begin{array}{c} \ b \xleftarrow{R} \{0, 1\}; (pk, sk) \xleftarrow{R} \text{kgen}; \\ (m_0, m_1, s) \xleftarrow{} A_1(pk); y \xleftarrow{} \text{enc}(m_b, pk); \\ b' \xleftarrow{} A_2(m_0, m_1, s, y) : b' = b \end{array} \right] - 1
\]

where \(A = (A_1, A_2)\) runs in time at most \(t\).

\(\text{AE}\) is IND-CPA if and only if \(\text{Succ}^{\text{ind-cca2}}_{\text{AE}}(t)\) is negligible when \(t\) is polynomial in the security parameter.
Suppose that $H$ is a hash function in the Random Oracle Model and that $f$ is a one-way trapdoor permutation. Consider the encryption function $E_{pk}(x) = f_{pk}(r) || H(r) \oplus x$, where $||$ denotes concatenation and $\oplus$ denotes exclusive or (Bellare & Rogaway, CCS’93).

- What is the decryption function?
- Show using CryptoVerif that this public-key encryption scheme is IND-CPA.
Exercise 3

Consider the fixed version of the Woo-Lam shared-key protocol, by Gordon and Jeffrey (CSFW’01):

\[
\begin{align*}
A &\rightarrow B: \quad A \\
B &\rightarrow A: \quad N \text{ (fresh nonce)} \\
A &\rightarrow B: \quad \{m3, B, N\}_{kAS} \\
B &\rightarrow S: \quad A, B, \{m3, B, N\}_{kAS} \\
S &\rightarrow B: \quad \{m5, A, N\}_{kBS}
\end{align*}
\]

At the end, \(B\) verifies that \(\{m5, A, N\}_{kBS}\) is the message from \(S\).

Show that, at the end of the protocol, \(A\) is authenticated to \(B\).

Suggestion: one may consider

1. First, a simple version in which \(A\) talks only to \(B\), \(B\) talks only to \(A\), and \(S\) talks only to \(A\) and \(B\).

2. Then, generalize to the case in which \(A\), \(B\), and \(S\) may also talk to dishonest participants.
Exercise 4

Consider the Needham-Schroeder public-key protocol, as fixed by Lowe. We first consider a simplified version without certificates:

\[
\begin{align*}
A \to B & : \{N_A, pk_A\}_{pk_B} \\
B \to A & : \{N_A, N_B, pk_B\}_{pk_A} \\
A \to B & : \{N_B\}_{pk_B}
\end{align*}
\]

Show that, at the end of the protocol, \(A\) and \(B\) are mutually authenticated.
Exercise 4

Now consider the full version with certificates:

\[
\begin{align*}
A \rightarrow S & : (A, B) \\
S \rightarrow A & : (pk_B, B, \{ pk_B, B \}^{sk_S}) \\
A \rightarrow B & : \{ N_A, A \}^{pk_B} \\
B \rightarrow S & : (B, A) \\
S \rightarrow B & : (pk_A, A, \{ pk_A, A \}^{sk_S}) \\
B \rightarrow A & : \{ N_A, N_B, B \}^{pk_A} \\
A \rightarrow B & : \{ N_B \}^{pk_B}
\end{align*}
\]

Show that, at the end of the protocol, \( A \) and \( B \) are mutually authenticated.
Exercise 5

A signature scheme $S$ consists of

- a key generation algorithm $(pk, sk) \leftarrow kgen$
- a signature algorithm $\text{sign}(m, sk)$
- a verification algorithm $\text{verify}(m, pk, s)$

such that $\text{verify}(m, pk, \text{sign}(m, sk)) = 1$.

The advantage of the adversary against unforgeability under chosen message attacks (UF-CMA) of signatures is:

$$\text{Succ}_{S}^{\text{uf\text{-cma}}} (t, q_{s}, l) =$$

$$\max_{\mathcal{A}} \Pr \left[ (pk, sk) \leftarrow kgen; (m, s) \leftarrow \mathcal{A}^{\text{sign}(\cdot, sk)}(pk) : \text{verify}(m, pk, s) \land m \text{ was never queried to the oracle } \text{sign}(\cdot, sk) \right]$$

where $\mathcal{A}$ runs in time at most $t$,
calls $\text{sign}(\cdot, sk)$ at most $q_{s}$ times with messages of length at most $l$.

Represent UF-CMA signatures in the CryptoVerif formalism.
Exercise 6

A public-key encryption scheme $AE$ consists of

- a key generation algorithm $(pk, sk) \xleftarrow{R} kgen$
- a probabilistic encryption algorithm $enc(m, pk)$
- a decryption algorithm $dec(m, sk)$

such that $dec(enc(m, pk), sk) = m$.

The advantage of the adversary against indistinguishability under adaptive chosen-ciphertext attacks (IND-CCA2) is

$$Succ^{ind-cca2}_{AE}(t, q_d) = \max_{A} 2 \Pr \left[ \begin{array}{l}
    b \xleftarrow{\{0, 1\}}; (pk, sk) \xleftarrow{R} kgen; \\
    (m_0, m_1, s) \leftarrow A_1^{dec(., sk)}(pk); y \leftarrow enc(m_b, pk); \\
    b' \leftarrow A_2^{dec(., sk)}(m_0, m_1, s, y) : b' = b \\
    A_2 \text{ has not called } dec(., sk) \text{ on } y
\end{array} \right] - 1$$

where $A = (A_1, A_2)$ runs in time at most $t$ and calls $dec(., sk)$ at most $q_d$ times. Represent IND-CCA2 encryption in the CryptoVerif formalism.
Exercise 6

- Represent INT-CTXT symmetric encryption in the CryptoVerif formalism. (See definition in Exercise 1.)
- Represent UF-CMA probabilistic MACs in the CryptoVerif formalism. (See definition in Exercise 1.)