Adaptive Security of Symbolic Encryption: the Case of Dynamic Corruptions.

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Joint work with Bogdan Warinschi

LSV - ENS Cachan

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Introduction

Symbolic and Computational Settings

1. Symbolic model: automatic verification.
2. Computational model: realism.

Computational soundness of symbolic encryption

Symbolic security $\Rightarrow$ Computational security.

Adversary Model

1. Passive adversary: eavesdropper.
2. Adaptive adversaries.
3. Active adversaries.

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Structure

1. Passive Soundness
2. Adaptive Soundness
3. Selective Decryption

Messages and Deductions

Messages

Messages are defined by the following grammar:

$$msg ::= (msg, msg) \mid \{msg\}_k \mid k \mid 0 \mid 1$$

Dolev-Yao Deduction System

Message m is deducible from the set of messages E, $E \vdash m$ if:

$$\frac{m \in E}{E \vdash m}\,(\text{Ax}) \qquad \frac{E \vdash (m_1, m_2)}{E \vdash m_1}\,(\text{Pr1}) \qquad \frac{E \vdash (m_1, m_2)}{E \vdash m_2}\,(\text{Pr2})$$

$$\frac{E \vdash m_1 \quad E \vdash m_2}{E \vdash (m_1, m_2)}\,(\text{P}) \qquad \frac{E \vdash m \quad E \vdash k}{E \vdash \{m\}_k}\,(\text{Ch}) \qquad \frac{E \vdash \{m\}_k \quad E \vdash k}{E \vdash m}\,(\text{Déc})$$

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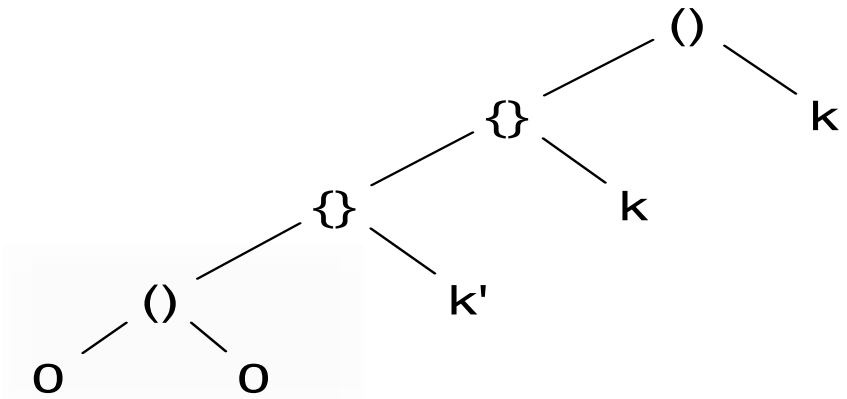
Accessible information from a message

Pattern

- $m = (\{\{0,0\}_{k'}\}_k, k)$
- Pattern : accessible information.



Equivalence

- $n = (\{\{1,1\}_{k'}\}_k, k)$
- m and n are indistinguishable.

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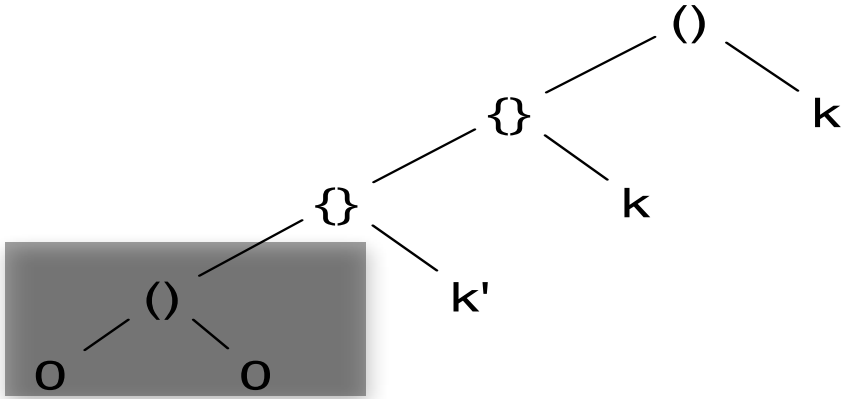

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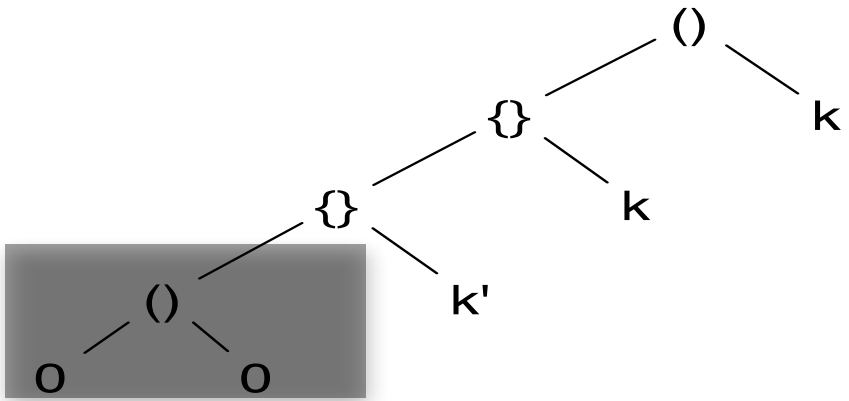

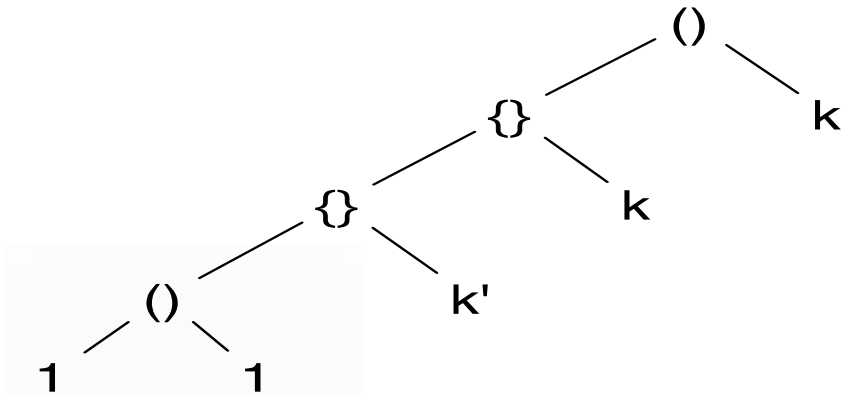

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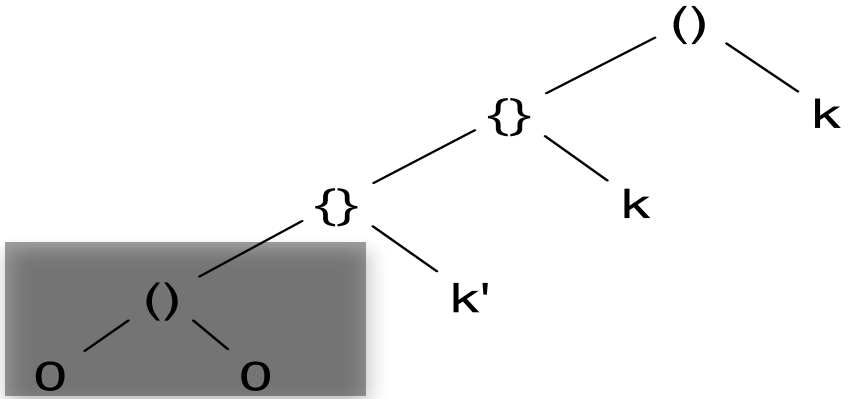

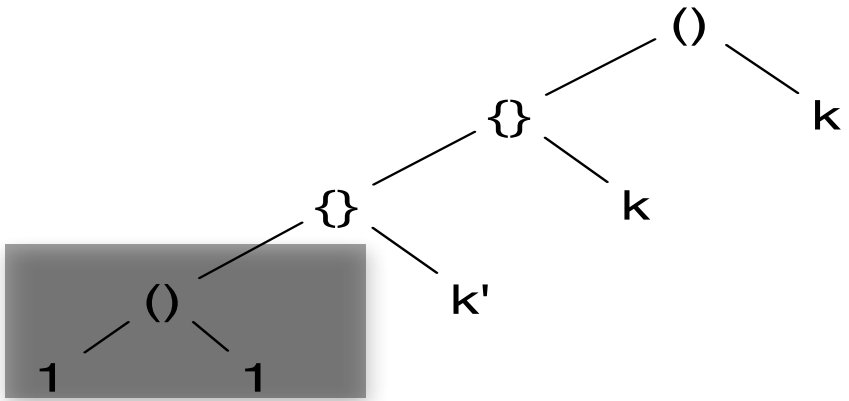

Symbolic Equivalence

Patterns

$$\begin{aligned}
\mathrm{pat}((m_1, m_2), K) &= (\mathrm{pat}(m_1, K), \mathrm{pat}(m_2, K)) \\
\mathrm{pat}(\{m'\}_k, K) &= \{\mathrm{pat}(m', K)\}_k && \text{if } k \in K \\
\mathrm{pat}(\{m'\}_k, K) &= \{\square\}_k && \text{if } k \notin K \\
\mathrm{pat}(k, K) &= k
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Symbolic Equivalence

- m is equivalent to n ($m \equiv n$) if

$$\mathrm{pat}(m, \{k / m \vdash k\}) = \mathrm{pat}(n, \{k / n \vdash k\})$$

- m is equivalent to n up to renaming ($m \cong n$) if

$$\exists \sigma \text{ permutation of keys}, m \equiv n\sigma$$

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Examples

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1. Key renaming:
$$k \cong k'$$
2. Impossible renaming:
$$(k, k') \ncong (k, k)$$
3. Encryption without key:
$$\{0\}_k \cong \{1\}_k$$
4. Key revelation:
$$(\{1\}_{k'}, \{0\}_k) \ncong (\{1\}_k, \{0\}_k)$$
5. Non-deterministic encryption:
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Semantic Security

Symmetric Encryption Scheme

A symmetric encryption scheme Π is made of three algorithms:

- $\mathcal{KG}$: key generation, $\eta \longrightarrow k$.
- $\mathcal{E}$: encryption, $bs, k \longrightarrow bs$.
- $\mathcal{D}$: decryption, $bs, k \longrightarrow bs$.

Security against Chosen Plaintext Attacks

Scheme $(\mathcal{KG}, \mathcal{E}, \mathcal{D})$ is IND-CPA secure if for any adversary $\mathcal{A}$,

$$Pr\left[\begin{array}{l} k \leftarrow \mathcal{KG} \\ \mathcal{A}^{(bs_0, bs_1) \rightarrow \mathcal{E}(\mathbf{bs_1}, k) = 1} \end{array}\right] - Pr\left[\begin{array}{l} k \leftarrow \mathcal{KG} \\ \mathcal{A}^{(bs_0, bs_1) \rightarrow \mathcal{E}(\mathbf{bs_0}, k) = 1} \end{array}\right]$$

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Computational Soundness

Indistingable Distributions

Two distributions D_0 et D_1 ($\eta \longrightarrow bs$) are indistinguishable, $D_0 \approx D_1$, if for any $\mathcal{A}$,

$$Pr\left[\begin{array}{l} bs \leftarrow \mathbf{D_1} \\ \mathcal{A}(bs) = 1 \end{array}\right] - Pr\left[\begin{array}{l} bs \leftarrow \mathbf{D_0} \\ \mathcal{A}(bs) = 1 \end{array}\right]$$

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Theorem (Soundness in the Passive Case)

If m and n are key acyclic and Π is IND-CPA, then

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Structure

1 Passive Soundness

2 Adaptive Soundness

3 Selective Decryption

The Adaptive Model

Model

- Intuition: $\mathcal{A}$ chooses (m_0^1, m_1^1) and obtains $[\![m_b^1]\!]_\Pi$
- Then $\mathcal{A}$ chooses (m_0^2, m_1^2) and obtains $[\![m_b^2]\!]_\Pi$...
- An adversary has to guess a bit b using an oracle $\mathcal{O}$.

$$\mathcal{O}_{b,\Pi,\eta} \; : \; (m_0, m_1) \rightarrow [\![m_b]\!]_\Pi$$

Definition

The advantage of $\mathcal{A}$ against Adaptive Security of Π is:

$$Adv^{ad}_{\mathcal{A},\Pi}(\eta) = Pr[\mathcal{A}^{\mathcal{O}_{1,\Pi,\eta}} = 1] - Pr[\mathcal{A}^{\mathcal{O}_{0,\Pi,\eta}} = 1]$$

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Easy ways to win:

1. $\mathcal{A}$ makes query $\mathcal{O}(0,1)$, receives b, $Adv^{ad}_{\mathcal{A},\Pi}(\eta) = 1$
   Restriction: queries have to be equivalent.
2. $\mathcal{A}$ makes query $\mathcal{O}(\{0\}_k, \{1\}_k)$, receives $\{b\}_k$,
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3. $\mathcal{A}$ makes query $\mathcal{O}(\{k\}_k, \{0\}_k)$ and receives b,
   Restriction: key-acyclicity of queries.

New possibilities of the adversary:

1. $\mathcal{A}$ makes query $\mathcal{O}(\{0\}_k, \{0\}_k)$, receives $\{0\}_k$,
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Computational Soundness (Micciancio-Panjwani'04)

Restrictions over adversaries

If $\mathcal{A}$ performs queries $(m_0^1, m_1^1), \ldots, (m_0^\ell, m_1^\ell)$, then

1. Requests must not contain any key cycles:
   $(m_0^1, ..., m_0^\ell)$ and $(m_1^1, ..., m_1^\ell)$ are acyclic
2. Requests must be symbolically equivalent:
   $(m_0^1, ..., m_0^\ell) \cong (m_1^1, ..., m_1^\ell)$
3. Keys are sent then used:
   $k \in \text{plaintext}(m_b^i) \Rightarrow k \notin \text{key}(m_b^j)$ for $j < i$

Theorem

If Π is IND-CPA secure then for any legal adversary $\mathcal{A}$,

$Adv^{ad}_{\mathcal{A},\Pi}$ is negligible in η

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Static Corruption Model

The send-then-use hypothesis

- Keys are sent then use.

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- Corrupt key k: $\mathcal{O}(k, k)$.
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Is it possible to weaken/suppress this hypothesis and still have computational soundness ?

Dynamic corruption model

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$$k \in \text{plaintext}(m_b^i) \Rightarrow k \notin \text{key}(m_b^j) \text{ for } j < i$$

- Use key k: $\mathcal{O}(\{0\}_k, \{0\}_k)$.
- Corrupt key k: $\mathcal{O}(k, k)$.
- send-then-use $\Rightarrow$ Static corruption model.

Is it possible to weaken/suppress this hypothesis and still have computational soundness ?

Dynamic corruption model

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Structure

1 Passive Soundness

2 Adaptive Soundness

3 Selective Decryption

The Selective Decryption Problem

Formulation of M. Naor, 1995

- Let Π be an IND-CPA secure encryption scheme.
- Generate $p(\eta)$ independent keys k_i using $\mathcal{KG}$.
- Distribute a CDROM with $p(\eta)$ images N_i encrypted with k_i.
- Clients can purchase k_i via a website.
- If a client purchase several keys k_i for $i \in I$,

  can he deduce anything on the N_j for $j \notin I$?

Intuitive answer: no because keys are independent.

Instance of a more complex problem: Selective Decommitment linked to other open problems in cryptography.

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In The Adaptive Model

Examples

We can map problems similar to selective decryption:

1. $\mathcal{A}$ makes query $\mathcal{O}\left(\{0\}_{k_1}, \{0\}_{k_1}\right)$, obtains bs_1,
2. $\mathcal{A}$ makes query $\mathcal{O}\left(\{0\}_{k_2}, \{0\}_{k_2}\right)$, obtains bs_2,
3. and so on ...
4. $\mathcal{A}$ makes query $\mathcal{O}(k_j, k_j)$ for every key in I,
5. $\mathcal{A}$ uses $\mathcal{O}$ to attack security of keys k_j for $j \notin I$.

Even problems close to adaptive selective decryption can be mapped.

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Solving Selective Decryption

Weakening of the send-then-use hypothesis

If a key k is corrupted then in previous messages,

- k has not been used to encrypt other keys – no key emission.
- k has not been used to encrypt any encryption –no nested encryptions.

Theorem

Let $\mathcal{A}$ be a legal adversary which exec. time is bounded by p,
Let $u(p)$ be the best possible adv ag IND-CPA for Π in time p,

$$\mathbf{Adv}^{ad}_{\mathcal{A},\Pi} \leq pu(p)$$

The same technique solves selective decryption and adaptive selective decryption.

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Nested Encryptions and Key Transmission

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If a key k is corrupted then in previous messages,

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- k has only been used to encrypt l nested encryptions: bounded nested encryptions.

$$\left\{\{\ldots\{0\}_{k_1}\ldots\}_{k_l}\right\}_k \text{ then } k \text{ can still be corrupted}$$

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Let $\mathcal{A}$ be a legal adversary which exec. time is bounded by p,

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Conclusion and Future Work

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- But bounded number of encryption layers
- Can be applied to the Active setting.

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1. Use static equivalence instead of patterns for symbolic equivalence for other primitives.
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